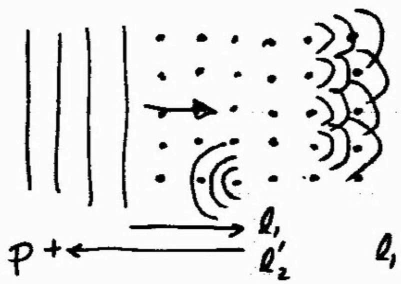
Reflection

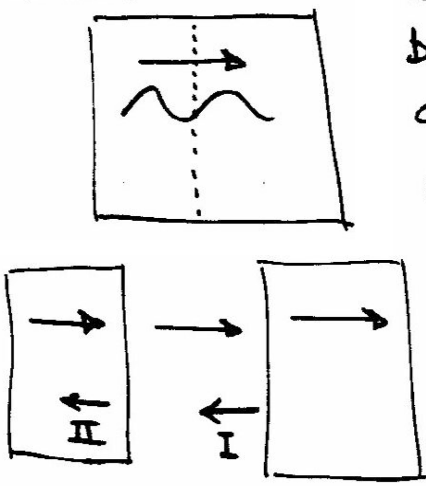
one point from earlier discussion:



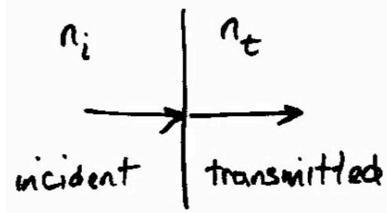
get constructively scattered wave only in forward direction; not in backward direc.!

$l_1 + l_2' \neq$ const.

- backward-going wave from one scatterer cancelled by waves from other (nearby) scatterers.
  - split a block:



reflected beams I and II; 180° out-of-phase.

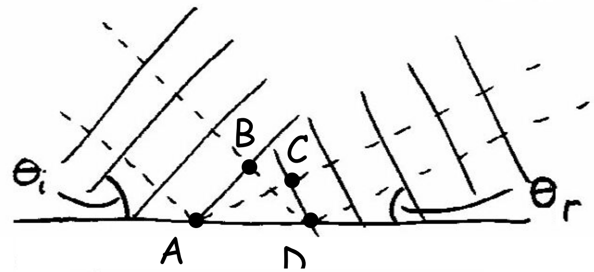


external reflec – $n_i < n_t$

internal reflec – $n_i > n_t$

Law of Reflection:

same time for scattered waves to reach reflected wavefront



$\phi = kx - \omega t$

$\Delta\phi = k\Delta x - \omega \Delta t = 0$

$\Rightarrow \Delta x = \frac{\omega}{k}\Delta t = v\Delta t = \text{const.}$

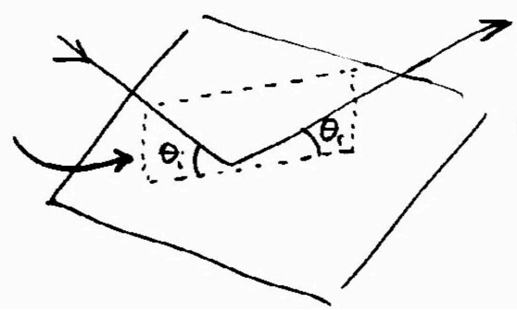
$\Rightarrow \overline{AC} = \overline{BD}$;

$\overline{AD}$ is common

$\Rightarrow \frac{\overline{BD}}{\sin\theta_i} = \frac{\overline{AC}}{\sin\theta_r}$

$\therefore \boxed{\theta_i = \theta_r}$

also, have common "plane of incidence"

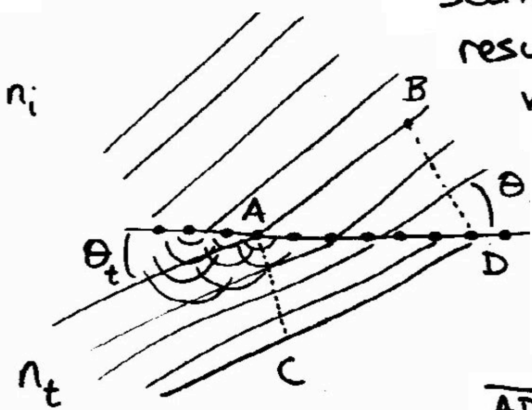


"specular reflection"

"rays" - lines everywhere perpendicular to wavefront (parallel to $\vec{S}$).

Refraction:

oscillators produce scattered waves; resultant has diff. velocity: $v = \frac{c}{n}$



$\Delta\phi = 0$

$\Rightarrow \Delta x = v_i \Delta t = \frac{c}{n_i}\Delta t$

$n_i \Delta x = \text{const.}$

$\overline{AD}$ is common

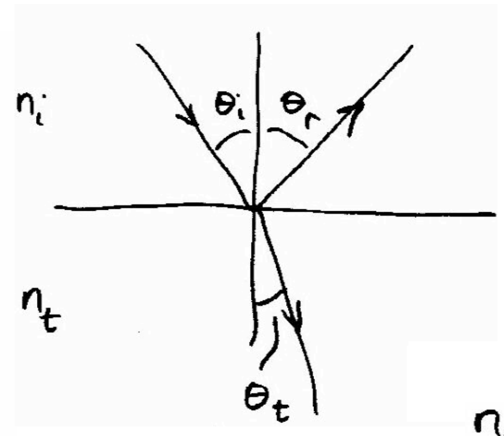
$$\Rightarrow \frac{\overline{BD}}{\sin\theta_i} = \frac{\overline{AC}}{\sin\theta_i}$$

and $\overline{BD} = v_i \Delta t$, $\overline{AC} = v_t \Delta t$

$$\Rightarrow \frac{v_i}{\sin\theta_i} = \frac{v_t}{\sin\theta_t}, \text{ or } \boxed{n_i \sin\theta_i = n_t \sin\theta_t}$$

Snell's Law.

- incident, reflected, and transmitted rays all lie in plane-of-incidence:



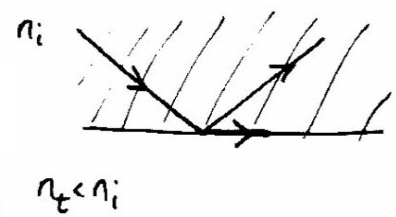
$n_{ti} \equiv \frac{n_t}{n_i}$ relative index of refraction

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n_{ti}}$$

$n_{ti} > 1$, $\theta_t < \theta_i$

$n_{ti} < 1$, $\theta_t > \theta_i$

what about when $\theta_t = 90°$?



$\theta_c \equiv \theta_i$,

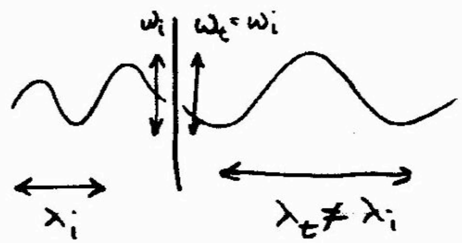
$n_i \sin\theta_c = n_t$

$\sin\theta_c = \frac{n_t}{n_i} = n_{ti}$

total internal reflection

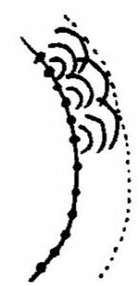
- for $\theta > \theta_c$, all energy in incident wave is transferred into reflected beam.

important point: $\omega = \text{const}$ in all waves; $v = \nu\lambda$ varies $\Rightarrow \lambda$ varies.



Prior treatment is example of:

Huygen's Principle: every point on wavefront (eg. in vacuum) is a point source of spherical waves.



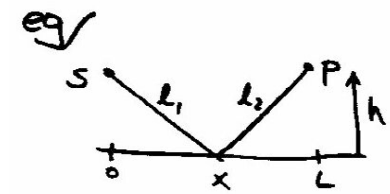
problem: produces backward going wave!

soln: derive rigorously, as Fresnel-Kirchoff integral.

Another method for solving this type of problem:

Fermat's Principle: light takes path of least time (actually, stationary with respect to variations) between to points.

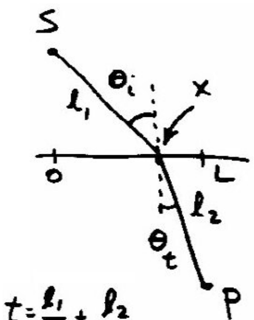
eg



$$t = \frac{l}{v}, \quad l = l_1 + l_2 = \sqrt{h^2+x^2} + \sqrt{h^2+(L-x)^2}$$

$$\frac{dl}{dx} = \frac{1}{2}(h^2+x^2)^{-1/2}2x + \frac{1}{2}(h^2+(L-x)^2)^{-1/2}2(L-x)(-) = 0$$

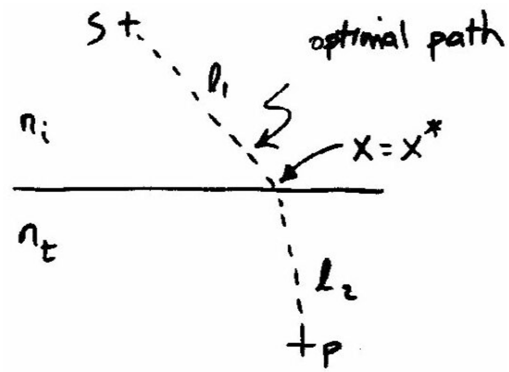
$$\Rightarrow \underline{x = \frac{L}{2}}$$



$$t = \frac{l_1}{v_i} + \frac{l_2}{v_t} = \frac{1}{c}(l_1 n_i + l_2 n_t)$$

$$\frac{dt}{dx} = 0 \Rightarrow \text{Snell's law.}$$

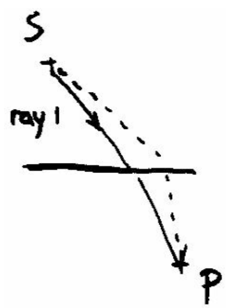
origin of Fermat's principle:

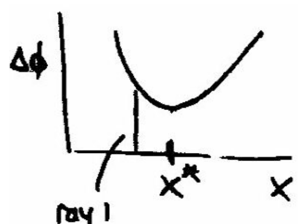


- optical path length OPL $= n\ell = n_i \ell_1 + n_t \ell_2$
- time to reach P, $t = \frac{1}{c} \cdot \text{OPL}$
- at particular time, $\Delta\phi_{SP} = \omega \sum \frac{\ell}{v}$

$$= \frac{\omega}{c} \cdot \text{OPL}$$

- say, path is nonoptimal one:

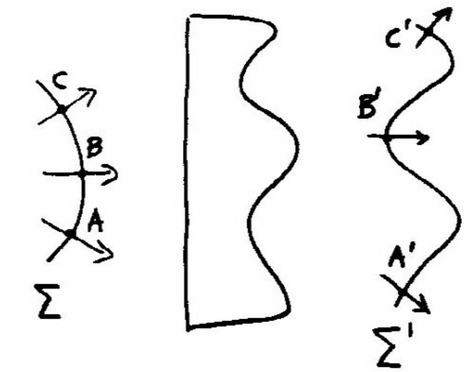




- if vary ray 1 path, $\Delta\phi$ varies linearly, and superposition $\rightarrow 0$ (consider OPL large).

- only at optimal path does the superposition yield nonzero result.

Light Rays and Wavefronts:



Σ, Σ' wavefronts, time Δt between them

A, A' corresponding points.

similarly B, B' ; C, C'

- group of rays which form a wavefront constitute a "normal congruence"

eg.

- this normal congruence is maintained after any number of reflections and refractions in isotropic materials.

$\Rightarrow$ ray tracing - follow rays thru optical system for fixed Δt, and form new wavefront.

Reflection and Transmission from Maxwell's Eqns (Fresnel Eqns.):

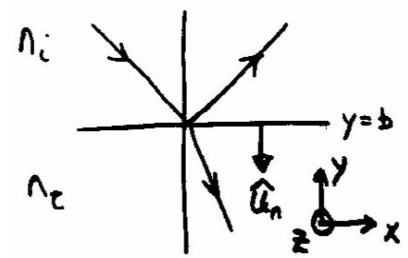
assume $\vec{E}_i = \vec{E}_{0i} \cos(\vec{k}_i \cdot \vec{r} - \omega_i t)$

take $\vec{E}_{0i} = \text{const}$ (plane polarized)

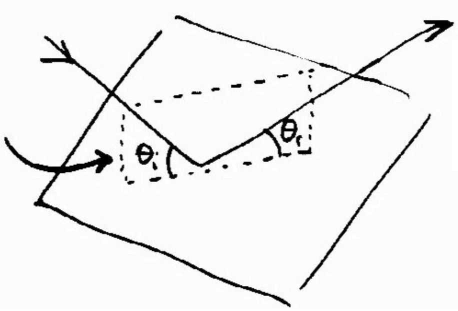
then can write

$$\vec{E}_r = \vec{E}_{0r} \cos(\vec{k}_r \cdot \vec{r} - \omega_r t + \varepsilon_r)$$

$$\vec{E}_t = \vec{E}_{0t} \cos(\vec{k}_t \cdot \vec{r} - \omega_t t + \varepsilon_t)$$



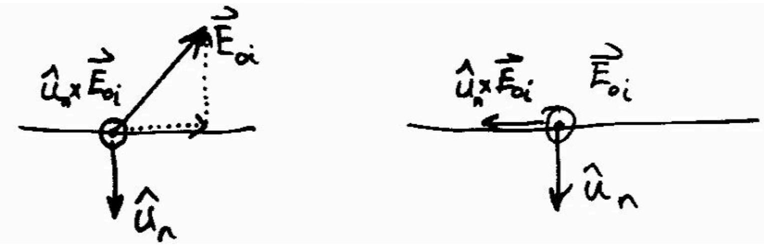
"plane of incidence"



ME $\Rightarrow$ tangential comp. of $\vec{E}$ continuous across interface

$\Rightarrow \hat{u}_n \times \vec{E}$ continuous

eg.



$$\Rightarrow \hat{u}_n \times \vec{E}_{0i} \cos(\vec{k}_i \cdot \vec{r} - \omega_i t) + \hat{u}_n \times \vec{E}_{0r} \cos(\vec{k}_r \cdot \vec{r} - \omega_r t + \varepsilon_r)$$

$$= \hat{u}_n \times \vec{E}_{0t} \cos(\vec{k}_t \cdot \vec{r} - \omega_t t + \varepsilon_t) \quad \underline{\underline{\text{at } y=b}}$$

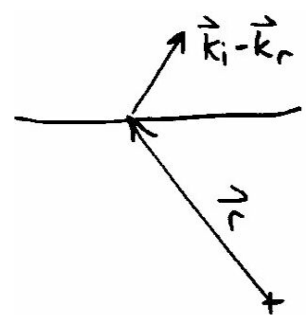
true for all $t \Rightarrow \omega_i = \omega_r = \omega_t \equiv \omega$

true for all (x, b, z)

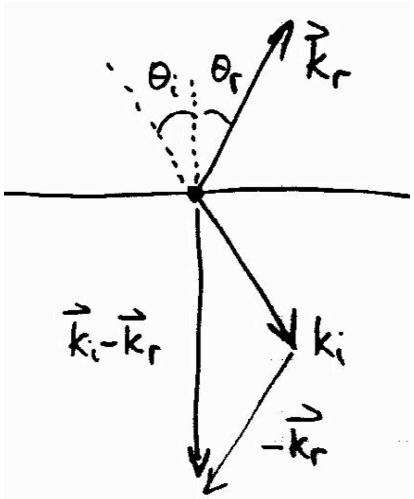
$$\Rightarrow (\vec{k}_i \cdot \vec{r} - \omega t)_{y=b} = (\vec{k}_r \cdot \vec{r} - \omega t + \varepsilon_R)_{y=b} = (\vec{k}_t \cdot \vec{r} - \omega t + \varepsilon_t)_{y=b}$$

$$\text{or} \quad \underbrace{(\vec{k}_i \cdot \vec{r})_{y=b} = (\vec{k}_r \cdot \vec{r} + \varepsilon_R)_{y=b}}_{} = (\vec{k}_t \cdot \vec{r} + \varepsilon_t)_{y=b}$$

$$(\vec{k}_i - \vec{k}_r) \cdot \vec{r} \,\Big|_{y=b} = \varepsilon_r$$



$\Rightarrow (\vec{k}_i - \vec{k}_r) \perp$ to interface



$\Rightarrow \; k_i \sin\theta_i = k_r \sin\theta_r$

but $k_i = k_r$ (same medium)

$\Rightarrow \; \boxed{\theta_i = \theta_r}$

<u>and</u>

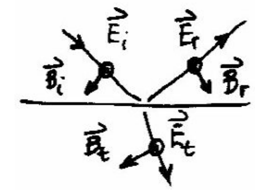
$$(\vec{k}_i - \vec{k}_t)\cdot\vec{r}\,\Big|_{y=0} = \varepsilon_t$$

$\Rightarrow \; \vec{k}_i - \vec{k}_t \perp$ to interface

$\Rightarrow \; k_i \sin\theta_i = k_t \sin\theta_t$

$v = \dfrac{\omega}{k} = \dfrac{c}{n} \;\Rightarrow\; \boxed{n_i \sin\theta_i = n_t \sin\theta_t}$

Now look at amplitudes:



<u>Case (i)</u>: $\vec{E} \perp$ to plane of incidence

$\Rightarrow \; \vec{B} \parallel$ " " " "

ME $\Rightarrow \; E_{oi} + E_{or} = E_{ot}$ $\quad (*)$

and, tangential comp. of $\vec{B}$ is continuous (take $\mu_t = \mu_i$)

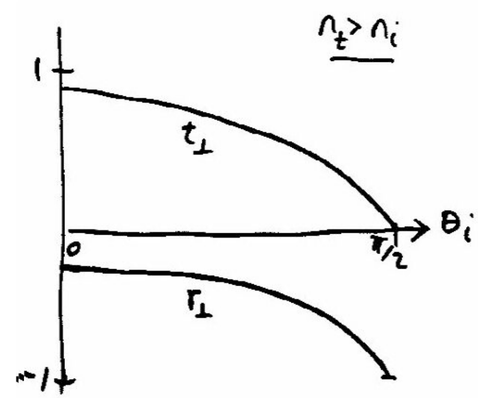
$$-B_{oi}\cos\theta_i + B_{or}\cos\theta_r = -B_{ot}\cos\theta_t$$

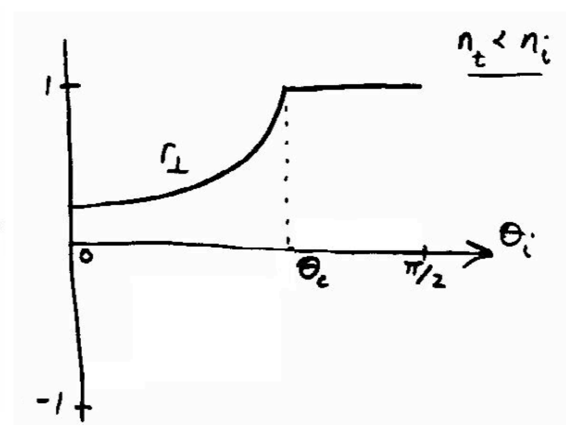
$B = \dfrac{E}{v} = \dfrac{En}{c} \quad \Rightarrow \; -E_{oi} n_i \cos\theta_i + E_{or} n_i \cos\theta_r = -E_{ot} n_t \cos\theta_t \quad (**)$

combine $(*)$ and $(**)$: $(\theta_r = \theta_i)$

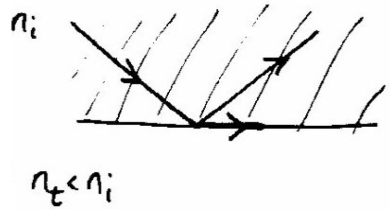
$$\left(\frac{E_{or}}{E_{oi}}\right)_\perp \equiv r_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$ amplitude reflec. coeff.

$$\left(\frac{E_{ot}}{E_{oi}}\right)_\perp \equiv t_\perp = \frac{2 n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$ amplitude trans. coeff.





θ_c - critical angle for total int. reflection

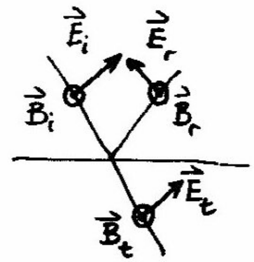


$$n_i \sin\theta_c = n_t$$

$$\sin\theta_c = \frac{n_t}{n_i} = n_{ti}$$

for $\theta_i > \theta_c$ treatment is not quite right (evanescent waves)

in general, $t_\perp - r_\perp = 1$.



Case (ii) $\vec{E}$ // to plane of incidence

$\Rightarrow \vec{B} \perp$ " " " "

$$\text{ME} \Rightarrow \quad E_{oi}\cos\theta_i - E_{or}\cos\theta_r = E_{ot}\cos\theta_t$$

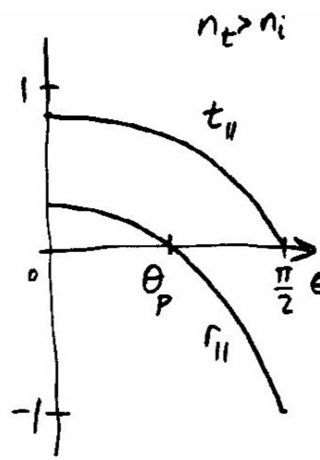
$$n_i E_{oi} + n_r E_{or} = n_t E_{ot}$$

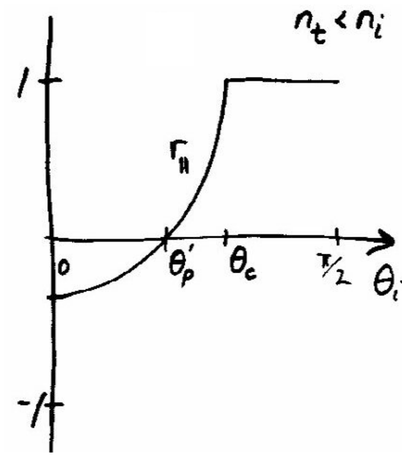
combine: $(\theta_r = \theta_i)$

$$\left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} \equiv r_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$\left(\frac{E_{ot}}{E_{oi}}\right)_{\parallel} \equiv t_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

(r's and t's can be further simplified using Snell's law).



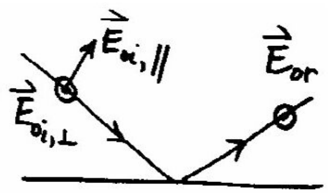


note: $t_\parallel - r_\parallel \neq 1$, but $t_\parallel + r_\parallel = 1$ at normal incidence

again, for $\theta > \theta_c$ treatment not quite right.

look at θ_P "Brewster's angle"

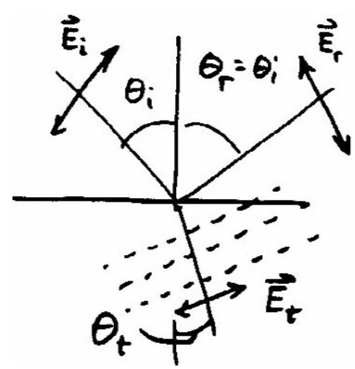
mixed polarized incident:



output only $\perp$ polarization

why?

oscillators don't emit $\perp$ to their oscillation direction:



if $\theta_t + \theta_r = \pi/2$

then $E_r = 0$.

$$\theta_t = \pi/2 - \theta_i$$

$$n_i \sin\theta_p = n_t \sin\theta_t$$
$$= n_t \cos\theta_p$$